Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 EXAMINATION – WINTER 2017

Subject Code: 110014 **Date:03 /01/2018**

Subject Name: Calculus

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (i) Is the following series convergent? Why? **04**

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots$$

(ii) Evaluate: **03**

$$\int_0^{\pi/2} sin^2x cos^4x\, dx.$$

(b) (i) Expand $cos\left(\frac{\pi}{4}+x\right)$ in powers of x. Hence, find the value of $\cos 46°$. **04**

(ii) Evaluate **03**

$$\lim_{x\to 0}\frac{1-cosx}{sinx}.$$

Q.2 **(a)** (i) If $u=log\left(\frac{x^2+y^2}{x-y}\right)$, then find the value of **04**

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}.$$

(ii) If $u=f(xy)$, show that **03**

$$x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=0.$$

(b) (i) Show that $f(x,y)=\begin{cases}\frac{x^2-y^2}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ is not continuous at $(0,0)$. **04**

(ii) If $x=rcos\theta$ and $y=rsin\theta$, find $\frac{\partial x}{\partial r}$ and $\frac{\partial r}{\partial x}$. **03**

Q.3 **(a)** (i) Find the maximum and minimum values and saddle point, if any, of **04**

$$f(x,y)=x^2+y^2+2x+4y+9.$$

(ii) Find the equation of tangent plane and normal line at a point $(1,2,3)$ to the surface $x^2+2y^2-3z=0$. **03**

(b) (i) Use Lagrange's method of multipliers to find the maximum value of $f(x,y,z)=xyz$ if it is given that $x+y+z=24$. **04**

(ii) Find the sum of the following series if it converges: **03**

$$1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots$$

Q.4 **(a)** (i) Expand $f(x)=sinx$ by Maclaurin's series. **04**

(ii) Test the convergence of the series **03**

$$\sum_{n=1}^{\infty}\frac{1}{n^2+1}.$$

(b) (i) Find the radius of convergence and interval of convergence of the series **04**

$$\sum_{n=0}^{\infty} \frac{x^n}{n\,!}.$$

(ii) Test the convergence of the series **03**

$$\sum_{n=1}^{\infty} \frac{n+1}{n^3+3}.$$

Q.5 **(a)** (i) Evaluate $\iint_R xydA$ over the area between $y = x^2$ and $y = x$. **04**

(ii) Use polar coordinates to evaluate **03**

$$\iint_R (x+y)dxdy$$

over the positive quadrant of the circle $x^2 + y^2 = a^2$.

(b) (i) Change the order of integration and evaluate: **04**

$$\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y} dxdy.$$

03

(ii) Evaluate:

$$\int_0^1 \int_0^y \int_0^{x+y} y\, dzdxdy.$$

Q.6 **(a)** (i) Trace the curve $r^2 = a^2 cos2\theta$. **04**

(ii) Use the fundamental theorem of integral calculus to find ${dy}/{dx}$ if **03**

$$y = \int_1^{x^2} cost\, dt.$$

(b) (i) Find the area of the circle $x^2 + y^2 = a^2$ by double integration. **04**

(ii) Find the volume of the region between the curve $y = \sqrt{x}, 0 \leq x \leq 3$ and the line $x = 3$ revolved about the x −axis to generate a solid. **03**

Q.7 **(a)** (i) Evaluate the improper integral $\int_0^1 \frac{dx}{\sqrt{x}}$. **04**

(ii) Evaluate: **03**

$$\lim_{x \to \infty} (x)^{\frac{1}{x}}.$$

(b) (i) Find the area of the region R enclosed by the lines $y = x, x = 1$ and the x −axis. **04**

(ii) If $x^2 + y^2 - xy = 0$, find $\frac{dy}{dx}$. **03**

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