Seat No.: ________ *Enrolment No.*___________

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER – I • EXAMINATION – WINTER • 2014

Subject code: 110008 **Date: 29-12-2014**

Subject Name: Mathematics - I

Time: 10:30 am - 01:30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1	**(a)**	(i) If $3-x^3 \le g(x) \le 3\sec x$, for all x, find $\lim_{x\to 0} g(x)$.	**02**
		(ii) Find the value of c so that function becomes continuous $f(x) = cx+1 \quad ; \ x \le 3$ $\quad\ \ = cx^2-1 \quad ; \ x > 3$	**02**
		(iii) Verify the Lagrange's mean value theorem for the function $f(x) = 2x^2 + 3x + 4$ in [a ,b] .	**03**
	(b)	(i) Find the Maclaurin series of function tanx upto terms containing x^5 .	**03**
		(ii) Evaluate using L' Hospital rule $\lim_{x\to 1}\left[\frac{1}{\log x} - \frac{x}{x-1}\right]$	**02**
		(iii) Evaluate using L' Hospital rule $\lim_{x\to 0} (a^x + x)^{1/x}$	**02**
Q.2	**(a)**	(i) Find the local maximum and local minimum value of the function $f(x) = x^3 - 9x^2 + 15x + 11$	**04**
		(ii) Evaluate $\int_0^\infty \frac{dx}{a^x + a^{-x}}$; a > 0	**03**
	(b)	(i) Trace the curve $x^3 + y^3 = 3axy$	**04**
		(ii) Discuss the convergence of the integral $\int_0^\infty \frac{1}{x^2}\,dx$	**03**
Q.3	**(a)**	(i) Does the sequence whose n^{th} term is $a_n = [(n+1)/(n-1)]^n$ converge? If so, find $\lim_{n\to\infty} a_n$	**04**
		(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n!}{n^n}$	**03**
	(b)	(i) Test the convergence of the series $\sum_{n=1}^{\infty}\left[\left(\frac{3}{4}\right)^{n-1} - \frac{4}{n(n+1)(n+2)}\right]$	**04**
		(ii) Show that the sequence [3/ (n+3)] is a decreasing sequence.	**03**
Q.4	**(a)**	(i) If $u = 2(ax+by)^2 - (x^2+y^2)$ and $a^2+b^2=1$ then prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$	**03**
		(ii) Find the equation of the tangent plane and normal line to the surface $2xz^2 - 3xy - 4x = 7$ at (1,-1,2)	**04**

(b) (i) Discuss the continuity of the function **03**

$$f(x,y) = (x^2 - y^2)/(x^2 + y^2) \quad ; \ (x,y) \neq (0,0)$$
$$= 0 \quad ; \ (x,y) = (0,0)$$

(ii) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then find the value of **04**

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$

Q.5 **(a)** (i) If $u = x^2 + y^2$, $x = a\cos t$, $y = b\sin t$ then find $\frac{du}{dt}$ **03**

(ii) Find $\frac{\partial(x,y,z)}{\partial(r,\theta,z)}$ where $x = r\cos\theta$, $y = r\sin\theta$ and $z = z$. **04**

(b) (i) Change the order of integration in the integral $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}\,dy\,dx$ and evaluate it. **03**

(ii) Find the volume of the region bounded by the surface $x = 0$, $y = 0$, $x + y + z = 1$ and $z = 0$. **04**

Q.6 **(a)** (i) Find the directional derivatives of $f(x,y,z) = x^2z + 2xy^2 + yz^2$ at the point $P(1,2,-1)$ in the direction of the vector $a = 2i+3j-4k$. **03**

(ii) Using Green's Theorem evaluate $\int_c (x^2y\,dx + x^2dy)$; where c is the boundary of the triangle whose vertices (0,0),(1,0), (1,1). **04**

(b) (i) Show that $F(x, y, z) = y^2z^3i + 2xyz^3j + 3xy^2z^2k$ is a conservative vector Field. **03**

(ii) If $F = 3xyi - y^2j$ then evaluate $\int_c F.dr$, where c is the arc of the parabola $y = 2x^2$ from (0, 0) to (1,2). **04**

Q.7 **(a)** (i) Test the convergence of the series **03**

$$\sum_{n=1}^{\infty}\frac{1}{n e^{n}}$$

(ii) If $u = f(x-y, y-z, z-x)$ then find $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}$ **04**

(b) (i) Evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dx\,dy$. **03**

(ii) Find the Taylor's series expansion of $f(x) = \sin x$ in the power of $(x - \pi/2)$. Hence obtain $\sin 91^{\theta}$. **04**

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