Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER 1st / 2nd (OLD) EXAMINATION WINTER 2016

Subject Code: 110008 **Date: 24/01/2017**

Subject Name: MATHS-1

Time:10:30 AM TO 1:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (i) If $\sqrt{5-2x^2} \le f(x) \le \sqrt{5-x^2}$ for $-1 \le x \le 1$ then find $\lim_{x \to 0} f(x)$ **03**

(ii) Verify Lagrange's Mean Value Theorem for the function $f(x) = x^2-2x+4$ on [1,5]. **04**

(b) (i) Using L'hospital Rule, Evaluate (1) $\lim_{x \to 0} \frac{1}{x^2}(1-x\cot x)$ **04**

(2) $\lim_{x \to 0} (\cot x)^{\sin x}$

(ii) Find Taylor's series generated by $f(x) = \frac{1}{x}$ at $a = 2$ **03**

Q.2 (a) (i) Trace the curve $r = a(1+\cos\theta)$; $a > 0$ **04**

(ii) Use the Fundamental Theorem of the integral calculus to find $\frac{dy}{dx}$ if $y = \int_{x}^{x^4} 3x\sin t \, dt$. **03**

(b) (i) Discuss the convergence of the integral $\int_{0}^{\infty} \frac{1}{x^2} dx$. **04**

(ii) Find the absolute maximum and minimum value of $f(x) = x^{2/3}$ on the interval [-2,3]. **03**

Q.3 (a) Discuss the convergence of the following series: **06**

(i) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+1}$ (ii) $\sum_{n=1}^{\infty} \frac{2n^2+3n}{5+n^5}$

(b) (i) Test the convergence of series $\sum_{n=1}^{\infty} \tan^{-1}\left(\frac{1}{n^2+n+1}\right)$ and if it is convergent then find its sum. **04**

04

(ii) Does the sequence whose nth term is $a_n = \left(\frac{n+1}{n-1}\right)^n$ converge? If so, then find $\lim_{n\to\infty} a_n$

Q.4 (a) **(i)** Discuss the continuity of the function **03**

$$f(x, y) = \frac{xy}{x^2+y^2} \quad ; \quad (x, y) \neq (0,0).$$
$$= 0 \quad ; (x, y) = (0,0)$$

(ii) If $u = \tan^{-1}\frac{x^3+y^3}{x-y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ **04**

(b) (i) Find the equation of tangent plane and the normal line to surface $x^2 + 2y^2 + 3z^2$=12 at (1,2-1) **04**

(ii) Find $\frac{dw}{dt}$ if w = xy + z , x = cost, y = sint , z = t **03**

Q.5 (a) (i) If x= rcosθ , y= rsinθ then evaluate $\frac{\partial(x,y)}{\partial(r,\theta)}$ **03**

(ii) Evaluate $\iint_R x\,dA$, where R is the region bounded by the Parabolas $y^2 = 4x$ and x^2=4y **04**

(b) (i) Evaluate : $\int_0^2\int_{y^2}^{2y} xy\,dx\,dy$ by changing order of integration. **04**

(ii) Evaluate: $\int_0^\pi\int_0^{a(1+\cos\theta)} r\,dr\,d\theta$ **03**

Q.6 (a) (i) Find the area enclosed by lemniscate $r^2 = a^2\cos 2\theta$ **04**

(ii) Evaluate $\iiint(x - 2y + z)$ dx dy dz over the region R, **03**
where R: 0≤ x ≤ 1 , 0 ≤ y ≤ x^2 , 0 ≤ z ≤ x+ y

(b) (i) Find the volume bounded by cylinder $x^2 + y^2 = 4$ and the plane $y + z = 3$ and $z = 0$ **04**

(ii) Evaluate $\int_0^1\int_0^1 dx\,dy$ by changing to polar co-ordinates. **03**

Q.7 (a) (i) Find the divergence and curl of $\bar{v} = (xyz)\hat{i}$+(3x^2y)$\hat{j}$+(x$z^2 - y^2$z)$\hat{k}$ at (2,-1,1) **04**

(ii) Evaluate $\int_C \bar{F}\cdot d\bar{r}$ along the parabola y^2=x between the points (0,0) and (1,1) where $\bar{F}$= $x^2\hat{i}$+ xy $\hat{j}$ **03**

(b) (i) Find the directional derivatives of $\phi = xy^2 + yz^2$ at the point (2,-1,1) in the direction of the vector $\hat{i} + 2\hat{j} + \hat{k}$. **04**

(ii) Evaluate $\oint_C [(x^2+2y)\, dx + (4x + y^2)]\, dy$ by Green's theorem where C is the boundary of the region bounded by y = 0,y = 2x and x + y = 3. **03**