Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER 1st / 2nd (OLD) EXAMINATION WINTER 2016

Subject Code: 110009 **Date: 20/01/2017**

Subject Name: Mathematics-II

Time:10:30 AM TO 1:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) 1. Solve the following system by Gauss elimination method: 03

$2x_1+2x_2+2x_3=0$, $-2x_1+5x_2+2x_3=1$, $8x_1+x_2+4x_3=-1$

2. Solve the system 04

$2x_1-x_2=\lambda x_1$, $2x_1-x_2+x_3=\lambda x_2$, $-2x_1+2x_2+x_3=\lambda x_3$ for x_1, x_2, x_3 in the two cases $\lambda=1$ and $\lambda=2$.

(b) 1. Solve the system 04

$-2y+3z=1$; $3x+6y-3z=-2$; $6x+6y+3z=5$ by Gauss-Jordan method.

2. Find the rank of the matrix $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$. 02

3 Prove that forthe matrix $A=\begin{bmatrix} -1 & 2+i & 5-3i \\ 2-i & 7 & 5i \\ 5+3i & -5i & 2 \end{bmatrix}$, iA is a skew Hermitian matrix. 01

Q.2 (a) 1. Find the vectors in R^2 with Euclidean norm 1 whose Euclidean inner product with $(3,-1)$ is zero. 0 3

2. State Cauchy-Schwarz inequality in R^n. Verity Cauchy-Schwarz inequality for the vectors $u=(0,-2,2,1)$ and $v=(-1,-1,1,1)$. Also, find $\|u-v\|$. 04

(b) 1. Let $T: R^3 \to R^3$ be a multiplication by . Determine whether T has an inverse. If so, find $T^{-1}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ where $A=\begin{bmatrix} 1 & 4 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 0 \end{bmatrix}$ 02

2. Show that the linear transformation $T: R^2 \to R^2$ defined by $T(x,y) = (2x - 6y, -5x + 15y)$ is not invertible. 02

3. Determine the algebraic and geometric multiplicity of the matrix 03
$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix}$

Q.3 (a) 1. Let R^3 have the Euclidean inner product. Transform the basis $u_1 = (1,1,1)$, $u_2 = (1,-2,1), u_3 = (1,2,3)$ into an orthogonal basisusing the Gram-Schmidt process. 03

2. Find the least square solution of the linear system $AX = b$ given by $x_1 - x_2 = 4$, $3x_1 + 2x_2 = 1$, $-2x_1 + 4x_2 = 3$ and find orthogonal projection of b on the column space of A. 04

(b) 1. Verify Caley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence find A^{-1} and A^4. 03

2. Reduce the quadratic form $Q(x,y,z) = 3x^2 + 3z^2 + 4xy + 8yz + 8xz$ to canonical form by linear transformation. 04

Q.4 (a) 1. Show that $S = \{1 - x - x^3, -2 + 3x + x^2 + 2x^3, 1 + x^2 + 5x^3\}$ is linearly independent in P_3. 03

2. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan elimination method. 04

(b) 1. Show that $S = \left\{ \begin{bmatrix} 1 & 2 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \right\}$ is a basis for M_{22}. 03

2. Find the matrix representation of quadratic form 02
$x^2 - 8y^2 + 14z^2 - 6xy + 10yz + 4xz$

3. Obtain the matrix of a linear transformation $T: R^2 \to R^3$ defined by the formula $T(x_1, x_2) = (x_1 + x_2, 2x_1 - x_2, 7x_2)$ with respect to the standard basis $B_1 = \{u_1, u_2\}$ and $B_2 = \{v_1, v_2, v_3\}$. 02

Q.5 (a) 1. Find the eigenvalues and eigenvectors of thematrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$. 04

		2.	Find matrix P that diagonalizes the matrix $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$. Also determine $P^{-1}AP$.	03
	(b)	1.	Let $T: R^2 \to R^3$ the linear transformation defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \left(\begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}\right)$. Find the matrix for the transformation T with respect to the basis $B_1 = \{u_1, u_2\}$ for R^2 and $B_2 = \{v_1, v_2, v_3\}$ for R^3, where $u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$, $u_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}$, $v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$	04
		2.	Consider the basis $S = \{v_1, v_2, v_3\}$ for R^3 where $v_1 = (1,1,1)$, $v_2 = (1,1,0)$ and $v_3 = (1,0,0)$ and let $T: R^3 \to R^3$ be the linear operator such that $T(v_1) = (2,1,-4)$, $T(v_2) = (3,0,1), T(v_3) = (-1,5,1)$. Find a formula for $T(x_1, x_2, x_3)$ and use the formula to find $T(2,4,-1)$.	03
Q.6	(a)	1.	Let V be the set of all ordered pairs of real numbers with vector addition defined as $(x_1 + y_1) + (x_2 + y_2) = (x_1 + x_2 + 1, y + y_2 + 1)$ show that first five axioms for vector addition are satisfied. Clearly mention the zero vector and additive inverse.	04
		2.	State Dimension Theorem and verify that for the given matrix s $A = \begin{bmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$	03
	(b)	1.	Let $S = \{(2,0,0), (0,2,0)\}$ show that S is not a basis for R^3 even if it is linearly independent.	03
		2	If $W = \{(x,y,z) / x - 3y + 4z = 0, x,y,z \in R\}$, prove that W is a sub-space of a vector space of R^3.	04
Q.7	(a)	1.	Consider a basis $S = \{v_1, v_2\}$ for R^2, where $v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$. Suppose w in R^2 has the coordinate vector $[w]_s = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$. Find w.	03
		2.	Let V be the space spanned by $v_1 = \sin x, v_2 = \cos x, v_3 = x$. Show that $S = v_1, v_2, v_3$ forms a basis for V.	01

3 Find a basis for column-space of the matrix $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ -2 & -4 & 4 & -7 \\ 1 & 2 & 1 & 2 \end{bmatrix}$. 03

(b) 1. Let $T : R^2 \to R^2$ be the linear operator defined by 04

$T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 - 2x_2 \\ -x_2 \end{bmatrix}$ and $B_1 = \{u_1, u_2\}$ and $B_2 = \{v_1, v_2\}$ where

$u_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, u_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, v_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$.

(i) Find the matrix T w.r.t. the basis B_1

(ii) Find the matrix T w.r.t. the basis B_2

2. Define: Symmetric matrix, Skew- Symmetric Matrix, Diagonal matrix 03

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