Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER–1/2 EXAMINATION – WINTER 2017

Subject Code: 110009 **Date: 30/12/2017**

Subject Name: Maths-II (entry year 2008-10 having backlog)

Time: 10:30 AM TO 01:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** Use Gauss elimination to solve the system of linear equations **07**

$$x_1 - 2x_2 - 3x_3 = 0, \qquad 2x_2 + x_3 = -8, -x_1 + x_2 + 2x_3 = 3$$

(b) Find the inverse of a matrix by using Gauss Jordan method **07**

$$\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Q.2 **(a)** Show that the set of all pairs of real numbers of the form $(1, y)$ is a vector space under the operations **07**

$(1, y_1) + (1, y_2) = (1, y_1 + y_2)$ and $k(1, y) = (1, ky)$.

(b) (1) Check whether the following are subspaces or not: **04**

(i) All vectors of the form $(a, b, 0)$, $V = R^3$

(ii) All 2×2 matrices A such that det(A) = 0

(2) Show that the vector $v = (1, 7, 3)$ is a linear combination of the vectors **03**
$v_1 = (1, -1, 2), v_2 = (0, 4, 2), v_3 = (-1, 5, 3)$.

Q.3 **(a)** Check for linear dependence independence of the following:

(1) (-3, 0, 4), (5, -1, 2), (1, 1, 3) **02**

(2) $2x^2 - x + 7, x^2 + 4x + 2, x^2 - 2x + 4$ **03**

(3) 1, e^x, e^{2x} **02**

(b) Determine the dimension and basis for the solution space of the following: **07**

$$x_1 - 3x_2 + x_3 = 0,$$
$$2x_1 - 6x_2 + 2x_3 = 0$$
$$3x_1 - 9x_2 + 3x_3 = 0$$

Q.4 **(a)** Verify rank nullity theorem for the matrix **07**

$$\begin{bmatrix} 1 & -1 & -1 \\ 4 & -3 & -1 \\ 3 & -1 & 3 \end{bmatrix}$$

(b) Let $\bar{u} = (u_1, u_2)$ and $\bar{v} = (v_1, v_2)$ be vectors in R^2. Verify that the inner product **07**

$$\langle \bar{u}, \bar{v} \rangle = 3u_1v_1 + 5u_2v_2$$

satisfies the four inner product axioms.

Q.5 **(a)** Let R^3 have the Euclidean inner product. Use Gram-Schmidt process to transform the basis $\{u_1, u_2, u_3\}$ into an orthonormal basis, where **07**

$$u_1 = (1, 1, 1), u_2 = (-1, 1, 0), u_3 = (1, 2, 1)$$

(b) Find the least squares solution of the system $AX = \boldsymbol{b}$ given by **07**
2x – 2y = 2, x + y = -1, 3x + y = 1.

Q.6 **(a)** Check for linear transformation **03**

(1) $T: R^2 \rightarrow R^2$ defined by $T(x, y) = (2x - y, x - y)$ **02**

(2) $T: R^2 \rightarrow R^3$ defined by $T(x, y) = (x, y + 2, x + y)$ **02**

(3) $T : M_{nn} \rightarrow M_{nn}$ defined by $T(A) = A^T$

(b) **07**

Let $T: R^2 \rightarrow R^3$ defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}$. Find the matrix for the transformation T with respect to the basis $B = \{u_1, u_2\}$ for R^2 and $B' = \{v_1, v_2, v_3\}$ for R^3 where $u_1 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, u_2 = \begin{bmatrix} 5 \\ 2 \end{bmatrix}, v_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$, $v_2 = \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$.

Q.7 **(a)** Find the eigenvalues and eigenvectors of the matrix **07**

$$\begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

(b) Verify Cayley-Hamilton theorem for the matrix A where **07**

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 1 & 3 & 3 \end{bmatrix}$$

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