Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

B. E. - SEMESTER –I • EXAMINATION – WINTER 2012

Subject code: 110015 **Date: 11-01-2013**

Subject Name: Vector Calculus and Linear Algebra

Time: 10.30 am – 01.30 pm **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) Convert the following matrix in to its equivalent Reduced Row Echelon form $\begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 & -1 \\ 0 & 0 & 5 & 10 & 0 & 15 & 5 \\ 2 & 6 & 0 & 8 & 4 & 18 & 6 \end{bmatrix}$. 04

(b) Attempt the following:

(1) Solve the system of equations by Gaussian elimination and back substitution. 03

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 9 \\ 2x_1 + 4x_2 - 3x_3 &= 1 \\ 3x_1 + 6x_2 - 5x_3 &= 0 \end{aligned}.$$

(2) Find the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$. 02

(c) Attempt the following:

(1) Find inverse of matrix $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ using row operation. 03

(2) Use Cramer's rule to solve 02

$$\begin{aligned} x_1 \qquad + 2x_3 &= 6 \\ -3x_1 + 4x_2 + 6x_3 &= 30 \\ -x_1 - 2x_2 + 3x_3 &= 8 \end{aligned}.$$

Q.2 (a) Find the rank and nullity of the matrix $\begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$. 04

(b) Attempt the following:

(1) Find the Eigenvalues of matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 2 & 0 \\ 12 & 15 & 3 \end{bmatrix}$ and hence deduce eigenvalues of A^5 and A^{-1}. 03

(2) Define Hermitian matrix and verify that the matrix 02

A = $\begin{bmatrix} 1 & 3+4i & -2i \\ 3-4i & 2 & 9-7i \\ 2i & 9+7i & 3 \end{bmatrix}$ is Hermitian matrix

(c) Attempt the following:

(1) Can vectors $u_1 = 3 + x^3, u_2 = 2 - x - x^2, u_3 = x + x^2 - x^3, u_4 = x + x^2$ form basis for vector space $P_3 = \{a + bx + cx^2 + dx^3 / a, b, c, d \in R\}$? 03

(2) Prove that the set of vectors $A = \{(x, y)/(x, y) \in R^2, x = 1\}$ can not be vector subspace of vector space R^2 under standard vector addition and scalar multiplication defined on R^2. 02

Q.3 (a) Expand the linearly independent set $S = \{(1,1,1,1), (1,2,1,2)\}$ to be a basis for R^4. 04

(b) Prove that the set of all positive real numbers forms vector space under the operations defined by, 05

Vector addition : $x + y = x.y$ and

Scalar multiplication: $\alpha.x = x^{\alpha}$ for all $x, y \in R^+$.

(c) Find the co-ordinate vector of $v = (5,-1,9)$ relative to the basis $S = \{v_1, v_2, v_3\}$ of R^3 where $v_1 = (1,2,1), v_2 = (2,9,0)$ and $v_3 = (3,3,4)$. 05

Also find the vector u in R^3 whose coordinate vector with respect to basis S is $(u)_S = (-1,3,2)$.

Q.4 (a) Let $A = \begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix}, B = \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix}$ be two matrices in real matrix space M_{22}. And let $\langle A, B \rangle = a_1b_1 + a_2b_2 + a_3b_3 + a_4b_4$. 04

Show that $\langle A, B \rangle$ is an inner product in M_{22}.

(b) Find a basis for the orthogonal complement of vector subspace $w = span\{v_1, v_2, v_3, v_4\}$ of vector space R^5, 05

where $v_1 = (2,2,-1,0,1), v_2 = (-1,-1,2,-3,1), v_3 = (1,1,-2,0,-1)$ and $v_4 = (0,0,1,1,1)$.

(c) Let R^3 have standard Euclidean inner product. Transform the basis $S = \{v_1, v_2, v_3\}$ in to an orthonormal basis using Gram-Schmidt process, where $v_1 = (1,0,0), v_2 = (3,7,-2)$ and $v_3 = (0,4,1)$. 05

Q.5 (a) Is the vector $\begin{bmatrix} 1 & 2 \\ 3 & 13 \end{bmatrix}$ in range of linear transformation $T : P_2 \to M_{22}$ defined by, $T(a + bt + ct^2) = \begin{bmatrix} a - 2b + c & -a + 3b \\ b + c & a + 2b + 5c \end{bmatrix}$. 04

(b) Find the associated matrix of the linear transformation $T : R^3 \to R^3$, $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_3 + x_1)$ with the basis $B = \{(1,0,0), (1,1,0), (1,1,1)\}$ and $B' = \{(1,0,1), (0,1,0), (1,0,-1)\}$ for the domain 05 and co-domain respectively.

(c) Attempt the following:

(1) Show that the Transformation, $T: R^3 \to R^3$ defined by $T(x_1, x_2, x_3) = (x_1 - x_2 + x_3, 3x_1 + x_2 - x_3, 2x_1 + 2x_2 + x_3)$ is linear. 03

(2) Using induced matrix associated with transformation, determine the new point after applying the transformation to the given point $x = (2, -6)$ rotated 30^0 in the counter clockwise direction. 02

Q. 6 (a) Find the Eigenvalues and Eigenvectors of Matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. 04

(b) Find a matrix P that diagonalizes matrix $A = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and hence determine $P^{-1}AP$. 05

(c) Reduce the quadratic form $2x_1^2 + 2x_2^2 + 2x_3^2 + 2x_1x_3$ in to canonical form. 05

Q. 7 (a) Attempt the following:

(1) Find the directional derivative of $f(x, y, z) = 2x^2 + 3y^2 + z^2$ at point P(2,1,3) in the direction of $\vec{a} = [1, 0, -2]$. 02

(2) Find divergence and curl of $\vec{v} = xyz[x, y, z]$. 02

(b) Verify Green's theorem for vector function $\vec{F} = (y^2 - 7y)\hat{i} + (2xy + 2x)\hat{j}$ and curve $C: x^2 + y^2 = 1$ 05

(c) Evaluate $\iint_s \vec{F} \cdot \hat{n}\, dA$ using divergence theorem. 05

Where $\vec{F} = [\cos y, \sin x, \cos z]$ and $S = \{(x, y, z) / x^2 + y^2 \le 4, |z| \le 2\}$.

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