Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER 1st / 2nd (NEW) EXAMINATION WINTER 2016

Subject Code: 2110015 **Date:20/01/2017**

Subject Name: Vector Calculus and Linear Algebra

Time:10:30 AM TO 1:30 PM **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1		**Objective Question (MCQ)**	**Mark**
	(a)	**Choose the appropriate answer for the following MCQs.**	**07**

1. Rank of 3×3 invertible matrix is

a) 1 b) 2 c) 3 d) 4

2. Let A be a Skew- Hermitian matrix then $A =$ __________

a) A^T b) $\left(\overline{A}\right)^T$ c) $-A^T$ d) $-\left(\overline{A}\right)^T$

3. The set $S = \{1, x, x^2, x^3\}$ span which of the following?

a) P_3 b) R c) R^3 d) M_{33}

4. Let $u = (1, -2)$ be a vector in R^2 with the Euclidean inner product, then $\|u\|$ is

a) 1 b) 5 c) $\sqrt{5}$ d) $\sqrt{3}$

5. If A is a matrix with 6 columns and $rank(A) = 2$, then $Nullity(A)$ is

a) 2 b) 4 c) 0 d) None of these

6. If $\lambda_1 = 2, \lambda_2 = 6$ are the eigen values of the matrix A, then the eigen values of A^T are

a) 2 & 6 b) $\frac{1}{2}$ & $\frac{1}{6}$ c) 4 & 36 e) None of these

7. The product of the eigen values of matrix $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ is,

a) -5 b) 10 c) 5 d) -10

(b) Choose the appropriate answer for the following MCQs. **07**

1. Let $I : V \to V$ be an identity operator, then $\ker(I)$ is,

a) V b) $\{0\}$ c) 0 d) None of these

2. The mapping $T : R^3 \to R^3$ defined by $T(x, y, z) = (x, y, 0)$ is called as

a) Projection b) Reflection

c) Rotation d) Magnification

3. If $f_1 = x$ and $f_2 = \sin x$, then Wronskian $W\left(\frac{\pi}{2}\right) =$ __________

a) $\frac{\pi}{2}$ b) 1 c) 0 d) -1

4. If $\vec{r} = xi + yj + zk$, then divergence of $\vec{r}$ is

a) 2 b) -2 c) 3 d) -3

5. The value of $curl(grad\,\phi)$, where $\phi = 2x^2 - 3y^2 + 4z^2$ is

a) $4xi - 6yj + 8zk$ b) $4x - 6y + 8z$ c) 6 d) $\vec{0}$

6. The weighted Euclidean inner product $\langle u, v\rangle = 3u_1v_1 + 2u_2v_2$ is the inner product on R^2 generated by

a) $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$ b) $\begin{bmatrix} \sqrt{3} & 0 \\ 0 & \sqrt{2} \end{bmatrix}$ c) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ d) $\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}$

7. If V is a finite-dimensional vector space, and $T : V \to V$ is a linear operator and $\ker(T) = \{0\}$, then

a) $R(T) \neq V$ b) T is one-to-one

c) $Nullity(T) \neq 0$ d)None of these

Q.2 (a) Convert the following matrix in to reduced row echelon form and hence find the rank of a matrix. **03**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$$

(b) Solve the following system of equations by Gauss Elimination method **04**

$$x_1 - 2x_2 + 3x_3 = -2$$

$$-x_1 + x_2 - 2x_3 = 3$$

$$2x_1 - x_2 + 3x_3 = -7$$

(c) (i) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ using Gauss Jordan Method **03**

(ii) For what choices of parameter the following system is consistent. **04**

$$x_1 + x_2 + 2x_3 + x_4 = 1$$
$$x_1 + 2x_3 = 0$$
$$2x_1 + 2x_2 + 3x_3 = \lambda$$
$$x_2 + x_3 + 3x_4 = 2\lambda$$

Q.3 **(a)** Consider the vectors $u = (1, 2, -1)$ and $v = (6, 4, 2)$ in R^3. Show that $w = (9, 2, 7)$ is a linear combination of u and v. **03**

(b) Determine a basis for and the dimension of the solution space of the homogeneous system **04**

$$2x_1 + 2x_2 - x_3 + x_5 = 0$$
$$-x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0$$
$$x_1 + x_2 - 2x_3 - x_5 = 0$$
$$x_3 + x_4 + x_5 = 0$$

(c) Show that the set of all pairs of real numbers of the form $(1, x)$ with the operations $(1, y) + (1, y') = (1, y + y')$ *and* $k(1, y) = (1, ky)$ is a vector space. **07**

Q.4 **(a)** Sketch the unit circle in an xy - coordinate system in R^2 using **03**

1. The Euclidean inner product $\langle u, v \rangle = u_1 v_1 + u_2 v_2$.
2. The weighted Euclidean inner product $\langle u, v \rangle = \frac{1}{9} u_1 v_1 + \frac{1}{4} u_2 v_2$.

(b) Attempt the following. **04**

1. Let $u = (u_1, u_2)$ and $v = (v_1, v_2)$ be vectors in R^2. Verify that the weighted Euclidean inner product $\langle u, v \rangle = 3u_1 v_1 + 2u_2 v_2$ satisfies the four inner product axioms.
2. Let R^4 have the Euclidean inner product. Find the cosine of the angle θ between the vectors $u = (4, 3, 1, -2)$ and $v = (-2, 1, 2, 3)$.

(c) Consider the vector space R^3 with the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectors $u_1 = (1,1,1)$, $u_2 = (0,1,1)$ & $u_3 = (0,0,1)$ into an orthogonal basis $\{v_1, v_2, v_3\}$; then normalize the orthogonal basis vectors to obtain an orthonormal basis $\{q_1, q_2, q_3\}$. **07**

Q.5 **(a)** Let $T : P_1 \to P_2$ be the linear transformation defined by $T(p(x)) = x(p(x))$. Find the matrix for T with respect to the standard bases $B = \{u_1, u_2\}$ and $B' = \{v_1, v_2, v_3\}$, where $u_1 = 1$, $u_2 = x$; $v_1 = 1$, $v_2 = x$, $v_3 = x^2$. **03**

(b) Express $\begin{bmatrix} 4 + 2i & 7 & 3 - i \\ 0 & 3i & -2 \\ 5 + 3i & -7 + i & 9 + 6i \end{bmatrix}$ as the sum of a Hermitian and a skew-Hermitian matrix. **04**

(c) State the Dimension theorem for Linear Transformation and find the rank and nullity **07**

of T_A, where $T_A : R^6 \to R^4$ be multiplication by

$$A = \begin{bmatrix} -1 & 2 & 0 & 4 & 5 & -3 \\ 3 & -7 & 2 & 0 & 1 & 4 \\ 2 & -5 & 2 & 4 & 6 & 1 \\ 4 & -9 & 2 & -4 & -4 & 7 \end{bmatrix}$$

Q.6 **(a)** Determine the algebraic and geometric multiplicity of $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. **03**

(b) For the matrix $A = \begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$, find the nonsingular matrix P and the diagonal matrix D such that $D = P^{-1}AP$. **04**

(c) Determine A^{-1} by using Cayley-Hamilton theorem for the matrix **07**

$A = \begin{bmatrix} 1 & 3 & 2 \\ 0 & -1 & 4 \\ -2 & 1 & 5 \end{bmatrix}$. Hence find the matrix represented by

$A^8 - 5A^7 - A^6 + 37A^5 + A^4 - 5A^3 - 3A^2 + 41A + 3I$

Q.7 **(a)** Show that $F = \left(y^2 - z^2 + 3yz - 2x\right)i + \left(3xz + 2xy\right)j + \left(3xy - 2xz + 2z\right)k$ is both solenoidal and irrotational. **03**

(b) Find the work done when a force $F = \left(x^2 - y^2 + 2x\right)\hat{i} - \left(2xy + y\right)\hat{j}$ moves a particle in the xy-plane from $(0,0)$ to $(1,1)$ along the parabola $y^2 = x$. Is the work done different when the path is the straight line $y = x$? **04**

(c) State Green's theorem and use it to evaluate the integral $\oint_C y^2 dx + x^2 dy$, where C is the triangle bounded by $x = 0,\ x + y = 1,\ y = 0$. **07**

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