Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2016

Subject Code: 110014 **Date: 02/06/2016**

Subject Name: Calculus

Time:02:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 (a) (i)Is the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}+......$ convergent? Does it converge absolutely? **04**

(ii) Evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^2 x \, dx$ **03**

(b) (i) Expand $\sin\left(\frac{\pi}{4}+x\right)$ in power of x. Hence find the value of $\sin 44^0$. **04**

(ii) Evaluate $\lim_{x\longrightarrow\frac{\pi}{2}} (\sec x - \tan x)$ **03**

Q.2 (a) (i) If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, show that $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=2\sin u\cos 3u$. **04**

(ii) If $u=f(x-y, y-z, z-x)$; show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$. **03**

(b) (i) Show that $f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}} & ; (x,y)\neq 0 \\ 0 & ; (x,y)\neq 0\end{cases}$ is continuous at origin. **04**

(ii) Evaluate the improper integral $\int_{-\infty}^{\infty}\frac{dx}{1+x^2}$. **03**

Q.3 (a) (i) Find the maximum and minimum values and saddle point of $f(x,y)=x^2+y^2+4x+6y+13$. **04**

(ii) Find the equation of plane and normal line at a point $(3,4,5)$ to the surface $x^2+y^2-4z=5$. **03**

(b) (i) Find a point on the plane $2x+3y-z=5$ which is nearest to origin, using Lagrange's method of undermined multipliers. **04**

(ii) Find the sum of the series if it converges $5-\frac{10}{3}+\frac{20}{9}-\frac{40}{27}+.........$ **03**

Q.4 **(a)** (ii) Expand $f(x) = e^{\sin x}$ by Maclurin's series. **04**

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2n^2 + 3n}{5 + n^5}$. **03**

(b) (i) Find the radius of convergence and interval of convergence of the series $\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}$. **04**

(ii) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^3 + 2}{2^n + 2}$. **03**

Q.5 **(a)** (i) Evaluate $\iint_R xy(x+y)dA$ over the area between $y = x^2$ *and* $y = x$. **04**

(ii) Evaluate $\iint_R (x^2 + y^2)xdxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$ by polar coordinates. **03**

(b) (i) Change the order of integration $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dxdy$. **04**

(ii) Evaluate $\int_0^1 \int_0^y \int_0^{x+2y} (x + y + z)dzdxdy$. **03**

Q.6 **(a)** (i) Trace the curve $r = a(1 + \cos\theta)$. **04**

(ii) Use the fundamental theorem to find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \; dt$. **03**

(b) (i) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **04**

(ii) Find the volume of the region between the curve $y = \sqrt{x}$, $0 \le x \le 4$ and the line $x = 4$ revolved about the $x-$axis to generate a solid. **03**

Q.7 **(a)** (i) Evaluate $\iint_R (x^2 - y^2)^2 dA$ over the area bounded by the lines $|x| + |y| = 1$ using transformations $x + y = u$; $x - y = v$. **04**

(ii) Evaluate $\lim_{x \to 0} \left(\frac{e^x + e^{2x} + e^{3x}}{3} \right)^{\frac{1}{x}}$ **03**

(b) (i) Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$. **04**

(ii) If $x^3 + y^3 - 3axy = 0$ find $\frac{dy}{dx}$. **03**

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