Seat No.: ________ Enrolment No.___________

# GUJARAT TECHNOLOGICAL UNIVERSITY

**BE- SEMESTER– 1st / 2nd • EXAMINATION – SUMMER 2016**

**Subject Code: 110015** **Date:30/05/2016**

**Subject Name: Vector Calculus and Linear Algebra**

**Time: 02:30 PM to 05:30 PM** **Total Marks: 70**

**Instructions:**

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

**Q.1** **(a)**

(i) Using Gauss-Jordan method find $A^{-1}$ for $A = \begin{bmatrix} 0 & 1 & -1 \\ 3 & 1 & 1 \\ 1 & 2 & -1 \end{bmatrix}$, if exists. **03**

(ii) Solve the linear system **04**

$3x_1 - x_2 + x_3 + 2x_4 = -2;$
$x_1 + 2x_2 - x_3 + x_4 = 1;$
$-x_1 - 3x_2 + 2x_3 - 4x_4 = -6;$

by Gauss elimination method.

**(b)** (i) Is the vector $v = (1,1)$ is a linear combination of the vectors $v_1 = (-2,4)$ and $v_2 = (3,-6)$? Justify. **03**

(ii) Determine whether the subset $S = \{(x_1, x_2, x_3) / x_1 + x_3 = -2\}$ of $\mathbb{R}^3$ is a subspace. **04**

**Q.2** **(a)**

(i) Find the rank of a matrix $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 3 & 4 & 1 & 2 \\ -2 & 3 & 2 & 5 \end{bmatrix}$. **03**

(ii) Determine whether the set of polynomial $\{2, x, x^2, 3x-1\}$ is linearly independent or linearly dependent. **04**

**(b)** (i) Find the angle between the two vectors $u = (2,-2,3)$ and $v = (-1,2,2)$. **03**

(ii) Determine whether $V = \mathbb{R}^2$ is an inner product space under the inner product $\langle u, v \rangle = u_1 v_1 - 2u_1 v_2 - 2u_2 v_1 + 3u_2 v_2$. **04**

**Q.3** **(a)** (i) Determine whether the mapping $T: \mathbb{R}^2 \to \mathbb{R}^2$, $T(x,y) = (e^x, e^y)$ is a linear or not. **03**

(ii) Determine whether the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2, T(x,y) = (4x - y, x)$ is one-to-one. **04**

**(b)** Let $V = \{(a,b) / a, b \in \mathbb{R}\}$. Let $v = (v_1, v_2)$ and $w = (w_1, w_2)$. Define $(v_1, v_2) + (w_1, w_2) = (v_1 + w_1 + 1, v_2 + w_2 + 1)$ and $k(v_1, v_2) = (kv_1 + k - 1, kv_2 + k - 1)$. Verify that $V$ is a vector space **07**

**Q.4** **(a)** (i) Let $V = P_2$ with inner product define by $\langle p, q \rangle = \int_0^1 p(x) q(x) dx$. Find $\langle p, q \rangle$ where $p(x) = 1 - x^2; q(x) = 1 - x + 2x^2$. **03**

(ii) Show that the set of polynomials $S = \{x^2 + 2x + 1, x^2 + 2, x\}$ spans the vector space $P_2$. **04**

**(b)** Verify the Green's theorem for $\oint_C (y^2 dx + x^2 dy)$ where $C$ is triangle bounded by $x = 0, x + y = 1$ and $y = 0$. **07**

**Q.5** **(a)** (i) If $r = xi + yj + zk$, Prove that $\nabla^2 r^n = n(n+1) r^{n-2}$. **03**

(ii) Find the directional derivative of $\phi = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of the line $PQ$ where $Q$ is the point $(5,0,4)$. **04**

**(b)** Find the matrix $P$ that diagonalizes the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 1 \\ 1 & -2 & 1 \end{bmatrix}$. **07**

**Q.6** **(a)** (i) If $u$ and $v$ are vectors in $\mathbb{R}^n$ then prove that $\|u+v\| \le \|u\| + \|v\|$. **03**

(ii) Show that $F = 2xyz\,i + (x^2z + 2y)\,j + x^2y\,k$ is conservative. Find its scalar potential function $\phi$. **04**

**(b)** Let $S$ be the basis for $\mathbb{R}^3$ given by $S = \{(1,1,1), (-1,1,0), (-1,0,1)\}$. Apply the Gram-Schmidt process to $S$ to find an orthonormal basis for $\mathbb{R}^3$. **07**

**Q.7** **(a)** (i) Let $V = \mathbb{R}^2$ with inner product defined by $\langle u, v \rangle = u_1v_1 + 3u_2v_2$. Let $u = (2,-2)$ and $v = (1,4)$. Verify that the Cauchy-Schwartz inequality is upheld. **03**

(ii) Find the basis for the row and column space of $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$. **04**

**(b)** Consider the basis $S = \{v_1, v_2, v_3\}$ for $\mathbb{R}^3$ where $v_1 = (1,1,1), v_2 = (1,1,0), v_3 = (1,0,0)$ and let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation such that $T(v_1) = (1,0), T(v_2) = (2,-1), T(v_3) = (4,3)$. Find a formula for $T(x_1, x_2, x_3)$ and use the formula to find $T(2,-3,5)$. **07**

*************