Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER 1st / 2nd EXAMINATION (OLD SYLLABUS) – SUMMER - 2017

Subject Code: 110015 **Date:29/05/2017**

Subject Name: Vector Calculus & Linear Algebra (VCLA)

Time: 2:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Attempt any five questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)** (1) Find rank of a matrix A= $\begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$ **03**

(2) Find inverse of A= $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 3 & 0 & 0 \\ 1 & 3 & 5 & 0 \\ 1 & 3 & 5 & 7 \end{bmatrix}$ using row operations. **04**

(b) (1) Solve the system by Gauss Elimination method: **04**
4x+y+2z = 12, 2x-3y+8z = 20, -x+11y+4z = 33

(2) Show that A= $\begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix}$ is orthogonal and find its inverse. **03**

Q.2 **(a)** Verify Green's theorem for $\oint_C [(y - \sin x)dx + \cos x dy]$: where C is the plane triangle enclosed by the lines y=0, x= $\pi/2, y = 2x/\pi$ **07**

(b) (1) Prove that vector F= $(y^2-z^2+3yz-2x)i + (3xz + 2xy)j + (3xy-2xz +2z)k$ is irrotational. **04**

(2) Find directional derivative of the function f(x,y,z)= $x^2+3y^2+z^2$ at the point P(2,1,3) in the direction of the vector i-2k **03**

Q.3 **(a)** Prove that the set R^+ of all positive real numbers with operations x+y = xy and kx= x^k is a vector space. **07**

(b) (1) Find the basis for V = span (S) for the subset S= { (1,2,3,-1,0),(3,6,8,-2,0),(-1,-1,-3,1,1),(-2,-3,-5,1,1) of R^5. **04**

(2) Express the polynomial x^2+4x-3 as a linear combination of x^2-2x+5 ,$2x^2-3x$, x+3. **03**

Q.4 **(a)** $T : R^4 \to R^3$ is a linear transformation defined by T(x, y, z, w) = (4x+y-2z-3w, 2x+y+z-4w, 6x-9z+9w).Find basis for the kernel and range of T and verify dimension theorem. **07**

(b) (1) Consider the basis S={u,v,w} for R^3 , where u = (1,2,1), v = (2,9,0) and w = (3,3,4). $T : R^3 \to R^2$ is a linear transformation such that T(u)=(1,0) , T(v)= (-1,1) and T(w)= (0,1).Find formula for T(x,y,z) and use it to find T(1,2,-1) **04**

(2) $T : R^3 \to R^3$ is a linear operator such that **03**

$T(x,y,z) = (3x+y, -2x-4y+3z, 5x+4y-2z)$. Show that T is one to one and also find $T^{-1}(x,y,z)$

Q.5 **(a)** Using Gram Schmidt process, construct an orthonormal basis for R^3 whose basis is the set {(2,1,3) ,(1,2,3) ,(1,1,1)} **07**

(b) (1) $f(t) = 4t + 1$ and $g(t) = 2t^2 + 1$ be the polynomial with inner product $\langle f, g\rangle = \int_0^1 f(t)g(t)\,dt$ .Find angle between f and g. **04**

(2) Verify Pythagorean theorem for the vectors u = (3,0,1,0,4,-1) and v = (-2,5,0,2,-3,-18) **03**

Q.6 **(a)** (1) Find eigen values and basis for eigen spaces of $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$ **04**

(2) Determine algebraic multiplicity of eigen values of $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **03**

(b) Verify Cayley Hamilton theorem for $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and find A^{-1} **07**

Q.7 **(a)** (1) Find unit normal vector to the surface $x^2+2y^2+z^2 = 7$ at (1,-1,2) **03**

(2) If $F = 3xyi - y^2j$, evaluate $\int_c F.dr$,where C is the arc of parabola $y=2x^2$ from (0,0) to (1,2) **04**

(b) (1) Determine whether the vectors x=(1,-2,3) y=(5,6,-1) z=(3,2,1) form linearly dependent set or linearly independent set. **04**

(2) Show that every square matrix can be expressed as the sum of symmetric and skew symmetric matrix. **03**

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