Seat No.: ________ **Enrolment No.___________**

GUJARAT TECHNOLOGICAL UNIVERSITY
BE- SEMESTER– 1st / 2nd (NEW) • EXAMINATION – SUMMER 2016

Subject Code: 2110015 **Date:30/05/2016**
Subject Name: Vector calculus & Linear Algebra
Time: 02:30 PM to 05:30 PM **Total Marks: 70**
Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Sixquestions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)** **Mark**

(a) **07**

1. Which of the following is orthogonal to $(1,2,-3)$?
(a) $(3,6,3)$ (b) $(-3,6,3)$ (c) $(-3,6,-3)$ (d) $(-3,-3,6)$

2. If $\lambda = 3,2$ are eigen values of 2×2 matrix A, then one of the eigen value of A^4 is
(a) 0 (b) 3 (c) 9 (d) 81

3. Which of the following is not a subspace of R^2?
(a) $\{0\}$ (b) line $y=5x$ (c) line $y=3x+2$ (d) R^2

4. Rank of the matrix $\begin{bmatrix} 5 & -3 & 4 \\ 0 & 2 & 9 \\ 0 & 0 & -6 \end{bmatrix}$ is
(a) 0 (b) 1 (c) 2 (d) 3

5. If A is an 5×6 matrix and rank of A is 4 then nullity of A is
(a) 0 (b) 1 (c) 2 (d) 3

6. If A is any square matrix then, $A+A^T$
(a) symmetric (b) skew symmetric (c) orthogonal (d) none of these

7. Which of the following is not an elementary matrix?
(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(b) **07**

1. If $r = xi + yj - zk$ then $curl(r)$ is
(a) 1 (b) 2 (c) 0 (d) none of these.

2. If $\phi = xyz$, then the value of $|grad\phi|$ at $(1,2,-1)$ is
(a) 0 (b) 1 (c) 2 (d) 3

3. The set $\{(0,0),(1,0)\}$ is
(a) linearly independent (b) linearly dependent
(c) basis of R^2 (d) none of these

4. If eigen values of a 3×3 matrix A are -1, 0, 1 the $trace(A)$ is
(a) 0 (b) 1 (c) -1 (d) none of these

5. Dimension of $P_3 = \{a+bx+cx^2+dx^3 : a,b,c,d \in R\}$ is
(a) 1 (b) 2 (c) 3 (d) 4

6. If u and v are nonzero orthogonal vectors in R^2 with Euclidian inner product then

(a) $\|u+v\|^2 = \|u\|^2 + \|v\|^2$ (b) $\|u+v\|^2 = 2\|u\|^2 + 2\|v\|^2$

(c) $\|u+v\|^2 = \|u\|^2 + 2\|v\|^2$ (d) $\|u+v\|^2 = 2\|u\|^2 + \|v\|^2$

7. If $\det A \neq 0$ then

(a) $AX = 0$ has no solution (b) $AX = 0$ has unique solution

(c) $AX = 0$ has infinitely many solution (d) none of these

Q.2 **(a)** Express $(5,-1,9)$ as a linear combination of $v_1 = (2,9,0)$, $v_2 = (3,3,4)$, $v_3 = (1,2,1)$. **03**

(b) Let $u = (u_1, u_2)$, $v = (v_1, v_2) \in R^2$. Check whether $\langle u, v \rangle$ defined as $\langle u, v \rangle = 4u_1v_1 + 6u_2v_2$ is an inner product on R^2? **04**

(c) Solve **07**

$$x_1 + x_2 + 2x_3 - 5x_4 = 3$$
$$2x_1 + 5x_2 - x_3 - 9x_4 = -3$$
$$2x_1 + x_2 - x_3 + 3x_4 = -11$$
$$x_1 - 3x_2 + 2x_3 + 7x_4 = -5$$

Using Gauss Jordan method.

Q.3 **(a)** Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ **03**

(b) Find the basis of column space of the matrix **04**

$$\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 1 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

Hence, find the rank of the matrix.

(c) Determine linear transformation $T : R^2 \to R^3$ such that $T(1,0) = (1,2,3)$ and $T(1,1) = (0,1,0)$. Also find $T(2,3)$ **07**

Q.4 **(a)** Check whether the function $T : R^2 \to R^2$ given by the formula $T(x, y) = (x + 2y, 3x - y)$ is linear transformation or not. **03**

(b) Check whether set of following matrices is linearly dependent? **04**

$$\left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix} \right\}.$$

(c) Show that the following set is basis for P_3. **07**

$$\left\{ 1 + 4x - 2x^2, 2x + x^2, -3 + x + x^2, 5 - 2x - 3x^3 \right\}$$

Q.5 **(a)** Find eigen values of $A = \begin{bmatrix} -5 & 4 & 34 \\ 0 & 0 & 4 \\ 0 & 0 & 4 \end{bmatrix}$. Is A invertible? **03**

(b) State why the following set are not vector space **04**

(i) $V = R^2$ with the operation

$(x_1, y_1) + (x_2, y_2) = (x_1 + y_1 + 1, x_2 + y_2 + 1)$

$k(x, y) = (kx, ky)$

(ii) $V = \{p \in P_2 : p(0) = 1\}$ with the usual operation.

(c) Find eigenvalues and basis for eigenspace for the matrix **07**

$$A = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix}.$$

Q.6 **(a)** Find $curl\,\mathrm{F}$, if $F = (y^2 \cos x + z^3)i + (2y \sin x - 4)j + 3xz^2 k$. Whether F is irrotational? **03**

(b) Find the directional derivative of $f(x, y, z) = x^3 - xy^2 - z$ at $(1,1,0)$ in the direction of $2i - 3j + 6k$ **04**

(c) For which value of "a" will the following system have **07**

(i) No solution?, (ii) Unique solution? (iii) Infinitely many solution.

$$x + 2y - 3z = 4$$
$$3x - y + 5z = 2$$
$$4x + y + (a^2 - 14)z = a + 2$$

Q.7 **(a)** Find the unit normal to the surface $z^2 = 4(x^2 + y^2)$ at a point $(1, 0, 2)$. **03**

(b) If $F = (2xy + z^3)i + x^2 j + 3xz^2 k$. Show that $\int_C F \cdot dr$ is independent of path of integration. Hence find the integral when C is any path joining $(1,-2,1)$ and $(3,1,4)$ **04**

(c) Verify Green's theorem for the function $F = (x + y)i + 2xyj$ and C is the rectangle in the $xy-$plane bounded by $x = 0, y = 0, x = a, y = b$. **07**

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