Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER 1st / 2nd EXAMINATION (NEW SYLLABUS) – SUMMER - 2017

Subject Code: 2110015 **Date:29/05/2017**

Subject Name: Vector Calculus & Linear Algebra

Time: 2:30 PM to 05:30 PM **Total Marks: 70**

Instructions:

1. **Question No. 1 is compulsory. Attempt any four out of remaining Six questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **Objective Question (MCQ)** **Mark**

(a) **07**

1. Let A be a non singular matrix of order $n \times n$ then $|adj\ A|$ is equal to
(a) 0 (b) 1 (c) 2 (d) $|A|^{n-1}$

2. Let A be a skew symmetric matrix of odd order then $|A|$ is equal to
(a) 0 (b) 1 (c) 2 (d) - 1

3. The maximum possible rank of a singular matrix of order 3 is
(a) 0 (b) 1 (c) 2 (d) 3

4. Let A be a square matrix of order n with rank r where r < n, then the number of independent solutions of the homogeneous system of equation AX = 0 is
(a) n (b) r (c) n - r (d) 1

5. The dimension of the polynomial space P_3 is
(a) 1 (b) 2 (c) 3 (d) 4

6. Let T: $R^2 \to R^2$ be a linear transformation defined by T(x, y) = (3x, 3y) then T is classified as
(a) Reflection (b) Magnification (c) Rotation (d) Projection

7. Let T: $R^3 \to R^3$ be a linear transformation defined by T(x, y, z) = (y, - x, 0) then the dimension of R(T) is
(a) 0 (b) 1 (c) 2 (d) 3

(b) **07**

1. The product of the eigen values of $\begin{bmatrix} 1 & 4 & 3 \\ 0 & 2 & 5 \\ 0 & 0 & 2 \end{bmatrix}$ is
(a) 1 (b) 2 (c) 3 (d) 4

2. Let A = $\begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$ then the eigen values of $A + 3I$ are
(a) 1, 3 (b) 2, 4 (c) 4, 6 (d) 2, 3

3. Let T: $R^2 \to R^2$ be a linear transformation defined by T(x, y) = (y, x) then it is
(a) One to one (b) Onto (c) Both (d) Neither

4. For vectors u and v, $\|u+v\|^2 - \|u-v\|^2$ is
(a) $< u, v >$ (b) $2 < u, v >$ (c) $3 < u, v >$ (d) $4 < u, v >$

5. If $\|u+v\|^2 = \|u\|^2 + \|v\|^2$, then the vectors u and v are
(a) Parallel (b) Orthogonal (c) dependent (d) Co linear

6. The magnitude of the maximum directional derivative of the function 2x + y + 2z at the point (1, 0, 0) is
(a) 0 (b) 1 (c) 2 (d) 3

7. For vector point function $\vec{F}$, divergence of $\vec{F}$ is obtained by
(a) $\nabla \cdot \vec{F}$ (b) $\nabla \times \vec{F}$ (c) $\nabla \vec{F}$ (d) $\nabla^2 \vec{F}$

Q.2 **(a)** Express the matrix $\begin{bmatrix} 4 & 2 & -3 \\ 1 & 3 & -6 \\ -5 & 0 & 7 \end{bmatrix}$ as a sum of a symmetric and a skew–symmetric matrix. **03**

(b) Find the inverse of $\begin{bmatrix} 2 & 1 & 3 \\ 3 & 1 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ using Gauss Jordan method **04**

(c) Determine the values of k, for which the equations **07**
$3x - y + 2z = 1, -4x + 2y - 3z = k,$ and $2x + z = k^2$
possesses solution. Find the solutions in each case.

Q.3 **(a)** Show (9, 2, 7) as a linear combination of (1, 2, - 1) and (6, 4, 2) **03**

(b) Find a basis for the null space of $\begin{bmatrix} 2 & -1 & -2 \\ -4 & 2 & 4 \\ -8 & 4 & 8 \end{bmatrix}$ **04**

(c) Check whether the set of all ordered pairs of real numbers (x, y) with the operations defined as $(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2)$ and $k(x, y) = (2kx, 2ky)$ is a vector space. If not, list all the axioms which are not satisfied. **07**

Q.4 **(a)** Check whether the vectors 1, sin^2x and $cox2x$ are linearly dependent or independent. **03**

(b) Find the eigen values and eigen vectors of $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$ **04**

(c) Verify Cayley Hamilton theorem for $\begin{bmatrix} 6 & -1 & 1 \\ -2 & 5 & -1 \\ 2 & 1 & 7 \end{bmatrix}$ and hence find A^{-1} and A^4 . **07**

Q.5 **(a)** Verify parallelogram law for $\begin{bmatrix} -1 & 2 \\ 6 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ 3 & 2 \end{bmatrix}$ under the Euclidean inner product on M_{22} **03**

(b) Let V be an inner product space. Prove that if u and v are orthogonal unit vectors of V then $\|u - v\| = \sqrt{2}$. **04**

(c) Let R^3 have Euclidean inner product. Transform the basis S = { u_1, u_2, u_3 } into an orthonormal basis using Gram Schmidt process, where $u_1 = (1, 0, 0)$, $u_2 = (3, 7, -2)$ and $u_3 = (0, 4, 1)$. **07**

Q.6 **(a)** Check whether T: $R^3 \to R^3$ defined by T(x, y, z) = (x + 3y, y, 2x + z) is linear. Is it one to one and onto ? **07**

(b) Let T: $R^2 \to R^3$ be a linear transformation defined by **07**
T(x, y) = (y, - 5x + 13y, - 7x + 16y). Find the matrix for T with respect to the bases $B_1 = \{ u_1, u_2\}$ for R^2 and $B_2 = \{ v_1, v_2, v_3 \}$ for R^3, where $u_1 = (3, 1)$, $u_2 = (5, 2)$, $v_1 = (1, 0, -1)$, $v_2 = (-1, 2, 2)$ and $v_3 = (0, 1, 2)$.

Q.7 **(a)** Find the directional derivative of $xy^2 + yz^3$ at the point (2, - 1, 1). **03**

(b) If $\vec{F} = (y^2 - z^2 + 3yz - 2x)i + (3xz + 2xy)j + (3xy - 2xz + 2z)k$ then show that $\vec{F}$ is both solenoidal and irrotational. **04**

(c) Verify Green's theorem for $\oint_C (3x - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the triangle with vertices (0, 0), (1, 0) and (0, 1). **07**

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