Seat No.: _____   Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY
Diploma Engineering - SEMESTER–I • EXAMINATION – SUMMER • 2015

Subject Code: 310034   **Date: 03-06-2015**

Subject Name: Mathematics - I

Time: 02:30 pm - 05:00 pm   **Total Marks: 70**

Instructions:
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. English version is considered to be Authentic.

Q.1 Fill in the blanks using appropriate choice from the given options.   **14**

(1) $\log_7 49 =$ _____
    (i) 7   (ii) 2   (iii) log7   (iv) log2

(2) $\log 1.\log 2.\log 3.\log 4 =$ _____
    (i) log24   (ii) log(1+2+3+4)   (iii) 1   (iv) 0

(3) For G.P series, $\dfrac{G}{a} = \dfrac{b}{G}$ then $G =$ _____
    (i) a+b   (ii) a-b   (iii) $\sqrt{ab}$ (iv) $(ab)^2$

(4) If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ then adjA = _____
    (i) $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ (ii) $\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$ (iii) $\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} -4 & -3 \\ -2 & -1 \end{pmatrix}$

(5) For 2×2 matrix, $A.A^{-1} =$ _____
    (i) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (ii) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (iii) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ (iv) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(6) 120 = _____ Radian.
    (i) $\dfrac{3\pi}{2}$ (ii) $\dfrac{5\pi}{2}$ (iii) $\dfrac{2\pi}{5}$ (iv) $\dfrac{2\pi}{3}$

(7) $\sin^2 55 + \sin^2 35 =$ _____
    (i) 1   (ii) -1   (iii) 0   (iv) 2

(8) $\cos(150) =$ _____
    (i) $\dfrac{\sqrt{3}}{2}$ (ii) $\dfrac{1}{\sqrt{2}}$ (iii) $-\dfrac{\sqrt{3}}{2}$ (iv) $\dfrac{1}{2}$

(9) The period of $\tan\dfrac{3x}{2} =$ _____
    (i) $\dfrac{3\pi}{2}$ (ii) $\dfrac{2\pi}{3}$ (iii) $\dfrac{4\pi}{3}$ (iv) 2π

(10) If $a = 2i - j + k$ then $|a| =$ _____
    (i) $\sqrt{2}$ (ii) $\sqrt{3}$ (iii) $\sqrt{6}$ (iv) $\sqrt{5}$

(11) If vectors x and y are perpendicular to each other then x.y=_____
    (i) 0   (ii) 1   (iii) -1   (iv) x×y

(12) If vector a= -i+6j+3k and b= -i+2j+4k then a.b=_____
    (i)(-1, 12 ,12) (ii)-1   (iii)25   (iv)12

(13) Evaluate: $\sin^{-1}\dfrac{1}{2}$ =_____

    (i) $\dfrac{\pi}{2}$ (ii) $\dfrac{\pi}{3}$ (iii) $\dfrac{\pi}{4}$ (iv) $\dfrac{\pi}{6}$

(14) Sin (A+B) =_____
(i)sinA+sinB (ii)sinAcosB+cosAsinB (iii) sinAsinB+cosAcosB (iv)sinAsinB

Q.2   **(a)**   Attempt Any Two   06

    (i) Prove that: $\log(\sqrt{x^2+1}+x) + \log(\sqrt{x^2+1}-x)= 0$

    (ii) Simplify: $\dfrac{1}{\log_{xy} xyz} + \dfrac{1}{\log_{yz} xyz} + \dfrac{1}{\log_{zx} xyz}$

    (iii) For G.P series $T_8 =243$ and $T_5 = 9$ then find a and r.

   (b)   Attempt Any Two   08

    (i) Find out addition of first 10 terms for series 2-4+8-16...........

    (ii) 9+99+999+...... find addition of first n terms.

    (iii) If $\log(\dfrac{a+b}{2}) = \dfrac{1}{2}(\log a + \log b)$ then prove that a=b.

Q.3   **(a)**   Attempt Any Two   06

    (i) Find constant term in expansion of $(2x^2 -\dfrac{1}{x})^6$

    (ii) Calculate approximate value of $\sqrt[3]{1003}$ using binomial theorem

    (iii) If $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ then prove that $A^2 -5A +7I = 0$

   (b)   Attempt Any Two   08

    (i) Find the solution of equations 3x-2y=8 , 5x+4y=6 using matrix method

    (ii) If $A= \begin{pmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{pmatrix}$ then prove that adjA=A.

    (iii) If $A= \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ and $B= \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ then find AB and BA.

Q.4   **(a)**   Attempt Any Two   06

(i)For what value of R, vectors (2,-3,5) and (R,-6,-8) becomes perpendicular to each other.

(ii) If a=(3,-1,-4) b=(-2,4,-3) and c=(-1,2,-1) then find $|3a-2b+4c|$.

(iii) If $A=\begin{pmatrix} 2 & 2 & 2 \\ 2 & 1 & -3 \\ 1 & 0 & 4 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 3 & 3 \\ 3 & 0 & 5 \\ 9 & 9 & -1 \end{pmatrix}$ and $C=\begin{pmatrix} 4 & 4 & 4 \\ 5 & -1 & 5 \\ -7 & 8 & -1 \end{pmatrix}$ then find

$2A-3B+C$.

(b)   Attempt Any Two   08

(i) If $x=(3,-1,2)$ and $y=(2,1,-1)$ then find unit vector perpendicular to given vectors.

(ii) Prove that angle between two vectors $(1,1,-1)$ and $(2,-2,1)$ is $\sin^{-1}\sqrt{\dfrac{26}{27}}$

(iii) The constant forces $(3,2,1)$ and $(1,5,2)$ acting on a particle. Under these forces a particle moves from the point $(1,3,-2)$ to $(3,1,4)$. Find the total work done.

Q.5   (a)   Attempt Any Two   06

(i) Simplify: $\dfrac{\cos(90^\circ-A)\cos(180^\circ-A)\tan(180^\circ+A)}{\sin(90^\circ+A)\sin(180^\circ-A)\tan(180^\circ-A)}$

(ii) Prove that: $\tan 55 = \dfrac{\cos10+\sin10}{\cos10-\sin10}$

(iii) Prove that: $\sin^{-1}x + \cos^{-1}x = \dfrac{\pi}{2}$, $|x|\leq 1$

(b)   Attempt Any Two   08

(i) Draw the graph of $y=\cos x$, where $0\leq x\leq \pi$.

(ii) Prove that: $\dfrac{1+\sin 2A-\cos 2A}{1+\sin 2A+\cos 2A} = \tan A$

(iii) Evaluate: $\cos\dfrac{19\pi}{6}\cdot\sin\dfrac{17\pi}{6} - \sin\dfrac{11\pi}{6}\cdot\cos\dfrac{13\pi}{6}$.

****

Q.1 સાચો વિકલ્પ પસંદ કરી ખાલી જગ્યા પૂરો.   14

(1) $\log_7 49 =$ _____

    (i) 7   (ii) 2   (iii) $\log 7$ (iv) $\log 2$

(2) $\log 1.\log 2.\log 3.\log 4 =$ _____

    (i) $\log 24$ (ii) $\log(1+2+3+4)$ (iii) 1   (iv) 0

(3) સ.ગુ. શ્રેણી માટે, $\dfrac{G}{a}=\dfrac{b}{G}$ હોય તો $G =$ _____

    (i) $a+b$ (ii) $a-b$ (iii) $\sqrt{ab}$ (iv) $(ab)^2$

(4) જો $A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ હોય તો $\mathrm{adj}A =$ _____

    (i) $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ (ii) $\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$ (iii) $\begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}$ (iv) $\begin{pmatrix} -4 & -3 \\ -2 & -1 \end{pmatrix}$

(5) 2×2 શ્રેણિક માટે, $A.A^{-1} =$ _____

    (i) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ (ii) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ (iii) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ (iv) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(6) $120^\circ =$ _____ રેડીયન

    (i) $\dfrac{3\pi}{2}$ (ii) $\dfrac{5\pi}{2}$ (iii) $\dfrac{2\pi}{5}$ (iv) $\dfrac{2\pi}{3}$

(7) $\sin^2 55 + \sin^2 35 = $ _____

   (i) 1   (ii) -1   (iii) 0   (iv) 2

(8) $\cos(150^\circ) = $ _____

   (i) $\dfrac{\sqrt{3}}{2}$ (ii) $\dfrac{1}{\sqrt{2}}$ (iii) $-\dfrac{\sqrt{3}}{2}$ (iv) $\dfrac{1}{2}$

(9) $\tan \dfrac{3x}{2}$ નુ આવર્તમાન = _____

   (i) $\dfrac{3\pi}{2}$ (ii) $\dfrac{2\pi}{3}$ (iii) $\dfrac{4\pi}{3}$ (iv) 2π

(10) જો a=2i −j+ k હોય તો , $|a| = $ _____

   (i) $\sqrt{2}$ (ii) $\sqrt{3}$ (iii) $\sqrt{6}$ (iv) $\sqrt{5}$

(11) જો સદિશો x અને y એક બીજાને લમ્બ હોય તો x.y= _____

   (i) 0   (ii) 1   (iii) -1   (iv) x × y

(12) જો સદિશો a= −i+6j+3k અને b= −i+2j+4k હોય તો a.b= _____

   (i)(-1, 12,12)   (ii)-1   (iii) 25   (iv)12

(13) કિમંત શોધો: $\sin^{-1} \dfrac{1}{2} = $ _____

   (i) $\dfrac{\pi}{2}$ (ii) $\dfrac{\pi}{3}$ (iii) $\dfrac{\pi}{4}$ (iv) $\dfrac{\pi}{6}$

(14) $\sin(A+B) = $ _____

   (i) sinA+sinB (ii) sinAcosB+cosAsinB (iii) sinAsinB+cosAcosB (iv) sinAsinB

Q.2 (અ)કોઇપણ બે ગણો.   06

   (i) સાબિત કરો કે: $\log(\sqrt{x^2+1} + x) + \log(\sqrt{x^2+1} - x) = 0$

   (ii) સાદુ રૂપ આપો: $\dfrac{1}{\log_{xy} xyz} + \dfrac{1}{\log_{yz} xyz} + \dfrac{1}{\log_{zx} xyz}$

   (iii) સ.ગુ શ્રેણી માટે, $T_8 = 243$ અને $T_5 = 9$ હોય તો a અને r શોધો

(બ) કોઇપણ બે ગણો .   08

   (i) 2- 4+8-16............ શ્રેણી માટે, પ્રથમ 10 પદોની સરવાળો શોધો .

   (ii) 9+99+999+...... પદોની સરવાળો શોધો.

   (iii) જો $\log(\dfrac{a+b}{2}) = \dfrac{1}{2}(\log a + \log b)$ હોય તો a=b સાબિત કરો.

Q.3 (અ) કોઇપણ બે ગણો.   06

   (i) $(2x^2 - \dfrac{1}{x})^6$ ના વિસ્તરણ માંથી અચલ પદ શોધો.

   (ii) દ્વિપદિ પ્રમેય ની મદદથી $\sqrt[3]{1003}$ નુ આસન્ન મુલ્ય શોધો.

   (iii) જો $A = \begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ આપેલ હોય તો સાબિત કરો કે $A^2 - 5A + 7I = 0$.

(બ) કોઇપણ બે ગણો.   08

   (i) 3x-2y=8, 5x+4y=6 સમીકરણો નો શ્રેણિક ની મદદથી ઉકેલ મેળવો.

(ii) જો $A = \begin{pmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{pmatrix}$ હોય તો સાબિત કરો કે adjA=A.

(iii) જો $A = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ અને $B = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ હોય તો AB અને BA શોધો.

Q.4 (અ)કોઇપણ બે ગણો.   06

(i) R ની કઇ કિમત માટે, સદિશો (2,-3, 5) અને (R,-6,-8) એક બીજાને લમ્બ થાય .

(ii) જો સદિશો a=(3,-1,-4) b=(-2,4,-3) અને c=(-1,2,-1) હોય તો $|3a - 2b + 4c|$ શોધો.

(iii) જો $A = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 1 & -3 \\ 1 & 0 & 4 \end{pmatrix}, B = \begin{pmatrix} 3 & 3 & 3 \\ 3 & 0 & 5 \\ 9 & 9 & -1 \end{pmatrix}$ અને $C = \begin{pmatrix} 4 & 4 & 4 \\ 5 & -1 & 5 \\ -7 & 8 & -1 \end{pmatrix}$ આપેલ શ્રેણિકો હોય તો,

2A-3B+C શોધો.

(બ) કોઇપણ બે ગણો.   08

(i) સદિશો $x = (3,-1,2)$ અને $y = (2,1,-1)$ ને લમ્બ એકમ સદિશ શોધો.

(ii) સાબિત કરો કે, સદિશો(1,1,-1) અને(2,-2,1) વચ્ચેનો ખુણો $\sin^{-1}\sqrt{\dfrac{26}{27}}$ છે.

(iii) જો કોઇ કણ પર બળો (3,2,1) અને(1,5,2) લાગે છે.આ બળો ની અસર હેઠળ કણ બિંદુ(1,3,-2) થી (3,1,4) સુધી જાય છે.તો થએલ કુલ કાર્ય શોધો.

Q.5(અ) કોઇપણ બે ગણો.   06

(i) સાદૂ રૂપ આપો: $\dfrac{\cos(90^{\circ} - A)\cos(180^{\circ} - A)\tan(180^{\circ} + A)}{\sin(90^{\circ} + A)\sin(180^{\circ} - A)\tan(180^{\circ} - A)}$

(ii) સાબિત કરો કે: $\tan 55^{\circ} = \dfrac{\cos 10^{\circ} + \sin 10^{\circ}}{\cos 10^{\circ} - \sin 10^{\circ}}$

(iii) સાબિત કરો કે: $\sin^{-1} x + \cos^{-1} x = \dfrac{\pi}{2}$, $|x| \le 1$

(બ) કોઇપણ બે ગણો.   08

(i) $y = \cos x$, જ્યા $0 \le x \le \pi$ માટે ગ્રાફ દોરો.

(ii) સાબિત કરો કે : $\dfrac{1+\sin 2A - \cos 2A}{1+\sin 2A + \cos 2A} = \tan A$

(iii) કિમત શોધો: $\cos \dfrac{19\pi}{6}.\sin \dfrac{17\pi}{6} - \sin \dfrac{11\pi}{6}.\cos \dfrac{13\pi}{6}$.

****