1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
Q. 1 (a) An electronic company is engaged in the production of two components $C_{1}$ and
$\mathrm{C}_{2}$ that are used in the radio sets. Each unit of $\mathrm{C}_{1}$ costs the company Rs. 5 in wages and Rs. 5 in material, while each of $\mathrm{C}_{2}$ costs the company Rs. 25 in wages and Rs 15 in material. The company sells both products on one period credit terms, but the company's labour and material expenses must be paid in cash. The selling price of $\mathrm{C}_{1}$ is Rs 30 per unit and of $\mathrm{C}_{2}$ it is Rs 70 per unit. Because of the company's strong monopoly in these components, it is assumed that the company can sell, at the prevailing prices, as many units as it produces. The company's production capacity is however limited by two considerations. First at the beginning of period 1, the company has an initial balance of Rs 4,000 second the company has available in each period 2,000 hours of machine time and 1,400 hours of assembly time. The production of each $\mathrm{C}_{1}$ requires 3 hours of machine time and 2 hours of assembly time, whereas the production of each $C_{2}$ requires 2 hours of machine time and 3 hours of assembly time. Formulate this problem as an LP model so as to maximize the total profit of the company.
(b) A manufacturer of jeans is interested in developing an advertising campaign that will reach four different age groups. Advertising campaigns can be conducted through TV, radio and magazines. The following table gives the estimated cost in paise per exposure for each age group according to the medium employed. In addition, maximum exposure levels possible in each of the media namely, TV radio and magazines are 40,30 and 20 millions, respectively. Also the minimum desired exposures within each age group namely 13-18, 19-25, 26-35 and older are $30,25,15$ and 10 millions. The objective is to minimize the cost of obtaining the desired minimum exposure level in each age group.

| Media | Age groups |  |  |  |
| :--- | :---: | :---: | :---: | :---: |
|  | $13-18$ | $19-25$ | $26-35$ | 36 and older |
| TV | 12 | 7 | 10 | 10 |
| Radio | 10 | 9 | 12 | 10 |
| Magazines | 14 | 12 | 9 | 12 |

Q. 2 (a) State the general rules for formulating a dual LP problem from its primal. Write the dual of the LP problem
Minimize $Z-x_{1}-3 x_{2}-2 x_{3}$
Subject to the constraints
$3 \mathrm{x}_{1}-\mathrm{x}_{2}+2 \mathrm{x}_{3}<=7$
$2 \mathrm{x}_{1}-4 \mathrm{x}_{2}>=12$
$-4 \mathrm{x}_{1}+3 \mathrm{x}_{2}+8 \mathrm{x}_{3}=10$
$\mathrm{X}_{1}, \mathrm{x}_{2}>=0 ; \mathrm{x}_{3}$ unrestricted in sign
(b) A salesman makes all sales in three cities $\mathrm{X}, \mathrm{Y}$ and Z only. It is known that he visits each city on a weekly basis and never visits the same city in successive weeks. If he visits city X in a give week, then he visits city Z in the next week. However, if he visits city Y or Z , he is twice as likely to visit city X than the other
city. Obtain the transition probability matrix. Also determine the proportionate visits by him to each of the cities in long run.

## OR

(b) An airline that operates 7 days a week has the timetable as given below. Crew must have a minimum layover of 5 hours between flights. Obtain the pairing of flights that minimize layover time away from home assuming that the crew can be based at either of the two cities. Suggest an optimum assignment of crew that result in small layover
Delhi - Jaipur

| Flight No. | Depart | Arrive |
| :---: | :---: | :---: |
| 1 | 7.00 am | 8.00 am |
| 2 | 8.00 am | 9.00 am |
| 3 | 1.30 pm | 2.30 pm |
| 4 | 6.30 pm | 7.30 pm |


| Jaipur - Delhi |  |  |
| :---: | :---: | :---: |
| Flight No. Depart <br> Arrive  <br> 101 8.00 am <br> $9 ; 15 \mathrm{am}$  <br> 102 $8 ; 30 \mathrm{am}$ <br> $1 ; 45 \mathrm{am}$  <br> 103 12 Noon <br> 1.15 pm  <br> 104 5.30 pm <br> 6.45 pm  |  |  |

Q. 3 (a) What is simulation? Describe its advantages in solving the problems. Give its main limitations with suitable example.
(b) Find the minimum value of objective function by graphical method
$Z=-x_{1}+2 x_{2}$
Subject to
$-\mathrm{x}_{1}+3 \mathrm{x}_{2}<=10$
$\mathrm{x}_{1}+\mathrm{x}_{2}<=6$
$x_{1}-x_{2}<=2$
$\mathrm{x}_{1}, \mathrm{x}_{2}>=0$

## OR

Q. 3 (a) Discuss Integer linear programming. Give an example each of a pure and a mixed integer linear programming problem.
(b) The BAC Ice cream company has a distribution depot in Janakpuri for distributing ice cream in Delhi. There are four vendors located in Delhi who have to be supplied ice cream every day. The following matrix displays the distance between the depot and the four vendors. What route should the company van follow that the total distance covered is minimum.

| From | To |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Depot | Vendor A | Vendor B | Vendor C | Vendor D |
| Depot | - | 3.5 | 3 | 4 | 2 |
| Vendor A | 3.5 | - | 4 | 2.5 | 3 |
| Vendor B | 3 | 4 | - | 4.5 | 3.5 |
| Vendor C | 4 | 2.5 | 4.5 | - | 4 |
| Vendor D | 2 | 3 | 3.5 | 4 | - |

Q. 4 (a) A rural clinic hires its staff from nearby cities and towns on a part time basis. The clinic attempts to have a general practitioner (GP) and an internist on duty during at least a portion of each week. The clinic has weekly budget of Rs. 1200. A GP charges the clinic Rs. 40 per hour, a nurse charges Rs 20 per hour and an internist charges Rs 150 . The clinic has established the following goals in order of priority.
(i) A nurse should be available for at least 30 hours per week.
(ii) The weekly budget of Rs 1,200 should not be exceeded.
(iii)A GP or internist should be available at least 20 hours per week.
(iv) An internist should be available at least 6 hours per week.

Formulate a goal programming model fro determining the number of hours to hire each staff member in order to satisfy the various goals.
(b) What is Degeneracy in Transportation problem solution? Explain the whole process of removing degeneracy from the solution.
Q. 4 (a) A foreign bank is considering opening a drive-in window for customer service. Management estimates that customers will arrive for service at the rate of 12 per hour. The teller, whom it is considering to staff the window can serve customers at the rate of one every three minutes. Assuming Poisson arrivals and exponential service, find
(i) Utilization of teller
(ii) Average number is system
(iii)Average waiting time in line
(iv) Average waiting time in system.
Q. 4 (b) The captain of a cricket team has to allot five middle batting positions to five batsmen. The average runs scored by each batsman at these positions are as follows:

|  | Batting position |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Batsman | I | II | III | IV | V |
| A | 40 | 40 | 35 | 25 | 50 |
| B | 42 | 30 | 16 | 25 | 27 |
| C | 50 | 48 | 40 | 60 | 50 |
| D | 20 | 19 | 20 | 18 | 25 |
| E | 58 | 60 | 59 | 55 | 53 |

Find the assignment of batsmen to position which would give the maximum number of runs.
Q. 5 (a) Explain the basic concepts of Sensitive Analysis. What are the different factors affecting the given solutions and how do we resolve them? Give a brief comment on each of them.
(b) Write the dual of the following problem. Graph both the Primal and Dual.

Maximize $z=5 x+7 y$
Subject to
$1 \mathrm{x}+\mathrm{y}<=4$
$3 \mathrm{x}+8 \mathrm{y}<=26$
$10 \mathrm{x}+7 \mathrm{y}<=35$
$\mathrm{X}, \mathrm{y}>=0$

## OR

Q. 5 (a) Explain the following concepts (with an example) in context to LPP.
a. Objective Function
b. Convex Polygon
(b) Solve minimum spanning tree

