

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY
MCA INTEGRATED- SEMESTER-II• EXAMINATION – SUMMER 2017

Subject Code: 4420601

Date: 31-05-2017

Subject Name: Discrete Mathematics for Computer Science (DMCS)

Time: 10.30am to 01.00pm

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

- Q.1 (a)** Define Join-irreducible elements & atom of the Boolean algebra. **07**
Draw Hasse diagram of (S_{210}, D) and determine Join-irreducible elements and atom of the Boolean algebra (S_{210}, D) .
- (b)** For the poset $\langle \{1\}, \{2\}, \{4\}, \{1,2\}, \{1,4\}, \{2,4\}, \{3,4\}, \{1,3,4\}, \{2,3,4\} \rangle, \subseteq$ **07**
Draw the Hasse diagram and find :
1) maximal elements and minimal elements
2) Greatest element and least element, if exists
3) Lower bounds of $\{1,3,4\}$ and $\{2,3,4\}$
4) Upper bounds of $\{2,4\}$ and $\{3,4\}$
- Q.2 (a)** (i) Show that in a lattice if $x \leq y \leq z$ then $x \oplus y = y * z$ **03**
(ii) $(a * b) \oplus (a * b') = a$ **02**
(iii) $a * (a' \oplus b) = a * b$ **02**
- (b)** Define Sublattice. **07**
Check whether the following are sublattice of $(S_{30}, \text{GCD}, \text{LCM})$ or not? Justify.
(i) $\{1,2,3,6\}, \text{GCD}, \text{LCM}$
(ii) $\{1,3,6,15\}, \text{GCD}, \text{LCM}$
- OR**
- (b)** Let (L, \leq) be a lattice. For any $a, b \in L$ prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$ **07**
- Q.3 (a)** Use the Quine - McCluskey method to simplify the sum-of-products expression: **07**
 $f(a,b,c,d) = \sum(10,12,13,14,15)$
- (b)** Define: Boolean Algebra. Find all Sub Boolean algebra of Boolean algebra $\langle S_{30}, \wedge, \vee, ', 0, 1 \rangle$ **07**
- OR**
- Q.3 (a)** Check whether (S_{45}, D) is a complemented lattice or not? Justify. **07**
- (b)** Use Karnaugh map representation to find a minimal sum-of-products expression of the following: **07**
(i) $f(a,b,c,d) = \sum(5,7,10,13,15)$
(ii) $f(a,b,c,d) = \sum(0,1,2,3,13,15)$
- Q.4 (a)** (i) Define: Group. Show that in a group $(G, *)$, **04**
for any $a, b \in G$ if $(a * b)^2 = a^2 * b^2$ then $(G, *)$ must be abelian.
(ii) If $\langle G, * \rangle$ be a group then for any two elements a and b of $\langle G, * \rangle$, prove that $(a * b)^{-1} = b^{-1} * a^{-1}$ **03**
- (b)** Define subgroup of a group, left coset of a subgroup $\langle H, * \rangle$ in the group $\langle G, * \rangle$. Find left cosets of $\{[0], [3]\}$ in the group $\langle \mathbb{Z}_6, +_6 \rangle$. **07**

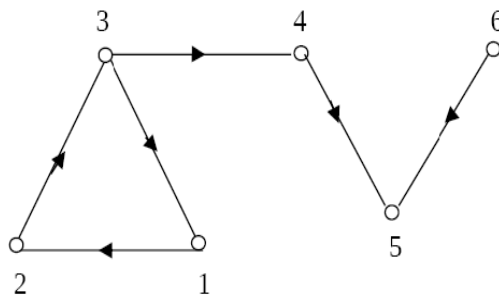
OR

- Q.4 (a)** Define Cyclic group. **07**
Show that $(\mathbb{Z}_4, +_4)$ is a cyclic group and find all the subgroups of $(\mathbb{Z}_4, +_4)$.

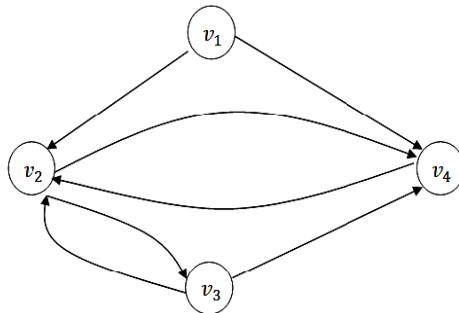
- (b)** Define “Group” “normal subgroup” of a group. Determine all the subgroups of the symmetric group $\langle S_3, \diamond \rangle$ given in the table below. **07**

\diamond	P1	P2	P3	P4	P5	P6
P1	P1	P2	P3	P4	P5	P6
P2	P2	P1	P5	P6	P3	P4
P3	P3	P6	P1	P5	P4	P2
P4	P4	P5	P6	P1	P2	P3
P5	P5	P4	P2	P3	P6	P1
P6	P6	P3	P4	P2	P1	P5

- Q.5 (a)** Define Nodebase. Find Nodebase of the following digraph. **07**

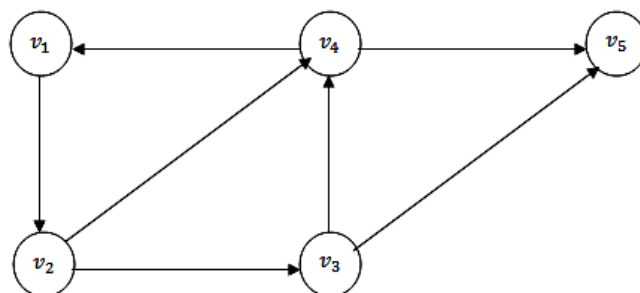


- (b)** Define Adjacency matrix. **07**
Obtain the adjacency matrix A of the digraph given below. Also find the elementary paths of lengths 1 and 2 from v_1 to v_4 .



OR

- Q.5 (a)** Define: Strong component, Unilateral component and weak component of the digraph. **07**
Determine Strong component, Unilateral component and weak component of the following digraph.



- (b)** Define Binary tree. Convert the given tree into binary tree $(v_0(v_1(v_2(v_3)(v_4))(v_5(v_6(v_7)(v_8)(v_9))(v_{10}(v_{11}(v_{12}))))))$ **07**
