Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

MCA – SEMESTER – III –EXAMINATION – WINTER - 2016

Subject Code:3630001 **Date:02/01/2017**

Subject Name: BASIC MATHEMATICS

Time:10.30 AM TO 01.00 PM **Total Marks: 70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**

Q.1 **(a)(i)** Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, $C = \begin{bmatrix} 7 & 1 \\ 2 & -5 \end{bmatrix}$, find AC, C^2 and BA. **03**

(ii) Define the transitive closure of a relation R in a set X. Find transitive closure of a relation $R = \{(a,b), (b,c), (c,a)\}$ defined on set $X = \{a, b, c\}$. **04**

(b) For the relations R and S over the set $\{1,2,3\}$, the relation matrices are given **07**

as $M_R = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $M_S = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$,

Find $M_{\bar{R}}$, $M_{\bar{S}}$, $M_{R \vee S}$ and $M_{\bar{S} \wedge \bar{R}}$

Check whether $M_{\bar{S} \wedge \bar{R}} = M_{R \wedge S}$ or not.

Q.2 **(a)(i)** (i) What is the universal quantification of the following? **04**

$x^2 + x$ is an even integer, where x is an integer.

Is the universal quantification a true statement?

(ii) Prove or disprove that the difference between two odd integers is an even integer.

(ii) Express the following using predicates, quantifiers and logical connectives. **03**

Also verify the validity of consequence:

Everyone who studies logic is good in reasoning.

Ajay is good in reasoning.

Therefore, Ajay studies logic.

(b) Let I be the set of integers and R be relation "congruence modulo 5". Show that R is an equivalence relation. Determine the R-equivalence classes generated by the elements of I. Show that set of equivalence classes forms a partition of I. **07**

OR

(b) Define (i) A compatibility relation (ii) A maximal compatibility block. Find maximal compatibility blocks of the relations shown in following figures. Also write their relation matrices. **07**

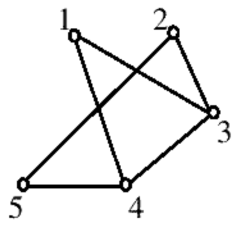


Figure-1

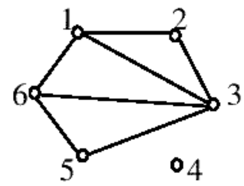


Figure-2

Q.3 **(a)(i)** Define characteristic function of a set. Show that, for all $x \in E$, **03**

$$\psi_{A \cap B}(x) = \psi_A(x) \times \psi_B(x)$$

Where A and B are any two subsets of a universal set E.

(ii) Describe any two hashing methods. **04**

(b) Define an inverse function. **07**

Let function $f : R \to R$ be given by $f(x) = x^3 - 2$.

Show that f^{-1} exists. Find f^{-1}.

OR

Q.3 **(a)(i)** In usual notation, Show that **03**

$$\psi_{A \cup B}(x) = \psi_A(x) + \psi_B(x) - \psi_{A \cap B}(x)$$

(ii) Describe any two collision resolution techniques for hashing functions. **04**

(b) Let $f : R \to R$ be given by $f(x) = -x^2$ and $g : R_+ \to R_+$ be given by $g(x) = \sqrt{x}$ where R_+ is the set of nonnegative real numbers and R is the set of real numbers. Find $f \circ g$. Is $g \circ f$ defined? Justify your answer. Determine whether (i) f is one-to-one (ii) g is one-to-one. **07**

Q.4 **(a)** (i) Show that for every $n \in N$, $n^3 + 2n$ is divisible by 3. **03**

(ii) Define a denumerable set. Show that the set of integers, positive, negative and zero is denumerable. **04**

(b) Define (1) Recursion (2) A primitive recursive function **07**

Show that the function $f(x, y) = x + y$ is primitive recursive.

OR

Q.4 **(a)** (i) Show that $n < 2^n$ for every $n \in N$. **03**

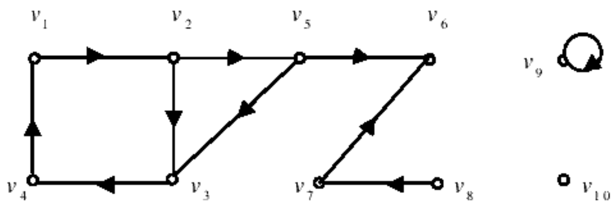
(ii) Show that the set $N \times N$ is denumerable. **04**

(b) Define (1) A primitive recursive relation (2)) A primitive recursive set **07**

Show that $\{(x, x) / x \in N\}$ which defines the relation of equality is primitive recursive relation.

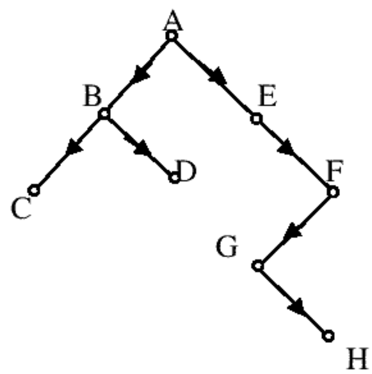
Q.5 **(a)** (i) Define Isomorphic graphs. What are the necessary conditions for two graphs to be isomorphic? Are they sufficient also? Justify your answer **04**

(ii) Define Reachable set of a node v . Find the reachable sets of (1) node v_1 and (2) node v_8 for the digraph given in following Figure. **03**



(b) Define (1) A leaf (2) A branch node of a directed tree. (3) A binary tree (4) A complete binary tree. **07**

Also write the order of nodes for the following tree, if it is traversed in (1) preorder (2) inorder (3) postorder



OR

Q.5 **(a)** (i) Define a directed tree. From the adjacency matrix of the simple digraph, how will you determine whether it is a directed tree? If it is a directed tree, how will you determine its root and terminal nodes? **04**

(ii) Describe link allocation techniques to represent binary trees. **03**

(b) Define (i) An unilateral component. (ii) A strong component of a digraph. **07**

Show that in a simple digraph $G(V, E)$,

(i) every node of the digraph lies in exactly one strong component.

(ii) every node and every edge lies in at least one unilateral component.

(iii) every edge and every edge lies in exactly one weak component.